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Oblique diffraction of surface waves by a submerged vertical plate

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Abstract.
A train of small-amplitude surface waves is obliquely incident on a fixed, thin, vertical plate submerged in deep water. The plate is infinitely long in the horizontal direction. An appropriate one-term Galerkin approximation is employed to calculate very accurate upper and lower bounds for the reflection and transmission coefficients for any angle of incidence and any wave number thereby producing very accurate numerical results.

1. Introduction

Mathematical study of water wave diffraction problems involving fixed vertical thin barriers within the framework of linearised theory of water waves was initiated long ago, using a variety of mathematical methods. For example, Dean [1] used the complex variable technique to study the reflection of a normally incident wave train by a fixed, thin, vertical barrier submerged in deep water; Ursell [2] used an integral equation procedure based on Havelock's [3] expansion of two-dimensional water wave potential to solve the complementary problem of a partially immersed barrier and obtained the closed form expressions for the velocity potential as well as the reflection and transmission coefficients. He also stated the results of the problem considered in [1]. Levine and Rodemich [4] used another integral equation procedure based on a suitable use of Green's integral theorem and Williams [5] used a reduction procedure to reinvestigate Ursell's [2] problem. Goswami [6] used an integral equation approach for the submerged barrier problem of Dean [1]. Also, Mandal and Kundu [7,8] demonstrated a number of mathematical methods in solving this problem. Again, Evans [9] earlier considered the problem of normal incidence on a thin plate submerged in deep water by using complex variable theory in conjunction with a Riemann-Hilbert boundary value problem and obtained the solution in closed form.

It should be noted that closed form solutions of the diffraction problems involving the aforesaid three configurations of the barrier only exist for the normal incidence of the wave train and deep water case, in the sense that the velocity potential as well as the reflection and transmission coefficients can be obtained exactly. However, for oblique incidence of the wave train, these problems cannot be solved in closed form, although the reflection and transmission coefficients can be obtained approximately. For example, Faulkner [10,11] used the Wiener-Hopf technique to study oblique water wave diffraction by a submerged plane vertical barrier as well as by a partially immersed vertical barrier and obtained the reflection coefficient in each case for large wave number. Jarvis and Taylor [12] pointed out an error of formulation in [11] involving the submerged barrier, which they corrected, thus obtaining asymptotically the reflection coefficient for large wave number by analysing an integral equation of the second kind with Cauchy kernel, to which the problem was reduced. The mathematical analysis in [10–

12] appears to be rather cumbersome. Later, Evans and Morris [3] utilized an approximate method involving the use of a one-term Galerkin approximation to the solutions of two appropriate integral equations to obtain very accurate upper and lower bounds for the reflection and transmission coefficients for all angles of incidence and wave numbers for the problem of oblique water wave diffraction by a fixed vertical barrier partially immersed in deep water. They also considered the complementary problem of a submerged barrier extending infinitely downwards by the same method, but did not present the results. Mandal and Goswami [14–16] utilized integral equation formulations for these problems based on appropriate use of Green's integral theorem and obtained the reflection and transmission coefficients approximately after solving the integral equations, also approximately, by a perturbation technique. In each case they presented graphical results for these coefficients for various values of the wave number and angle of incidence upto 15°. However, for an angle of incidence greater than 15° it is obvious that more terms were needed in the approximate series solutions of the integral equations concerned, and this somewhat restricts the use of this method as the analytical calculations are rather long and tedious.

Recently Mandal and Dolai [17] used the idea of Evans and Morris [13] to obtain very accurate lower and upper bounds for the reflection and transmission coefficients in the problems of water wave diffraction by four different structures involving a plane vertical barrier present in water of uniform finite depth. These structures consist of an immersed plate, a submerged plate extending down to the bottom, a barrier with a submerged gap and a submerged plate which does not extend to the bottom.

The problem of oblique water wave diffraction by a plate submerged in deep water considered in [16] is reinvestigated in this paper by using the method utilized in [13] for the problem of a partially immersed vertical barrier. The reflection and transmission coefficients are obtained in terms of integrals involving the unknown horizontal component of velocity across the gaps above and below the plate, and the difference of velocity potential across the plate. These unknown functions satisfy certain integral equations. The known exact solutions for normally incident waves are utilized as a one-term Galerkin approximation to the solutions of the appropriate integral equations to obtain very accurate upper and lower (numerical) bounds for the reflection and transmission coefficients for all angles of incidence and wave numbers. This method of obtaining very accurate upper and lower bounds to produce very accurate numerical results, appears to be simple in comparison to the method used in [16] for this problem. The reflection coefficient is depicted graphically for various values of the wave number and the angle of incidence.

2. Formulation of the problem

A train of progressive waves represented by the velocity potential

$$\psi_0(x, y, z, t) = \mathrm{Re}\{\exp(-Ky + i\mu x + i\nu z - i\sigma t)\}$$

where $\mu = K\cos\alpha, \nu = K\sin\alpha, K = \sigma^2/g, g$ is the acceleration due to gravity, σ is the circular frequency. The wave train is assumed to be obliquely incident on a thin vertical plate occupying the position $x = 0, y \in S$ and $-\infty < z < \infty$, where $S = (a, b)$, at an angle α to the normal of the plate from negative infinity. A sketch of the geometry of the problem is depicted in Figure 1.

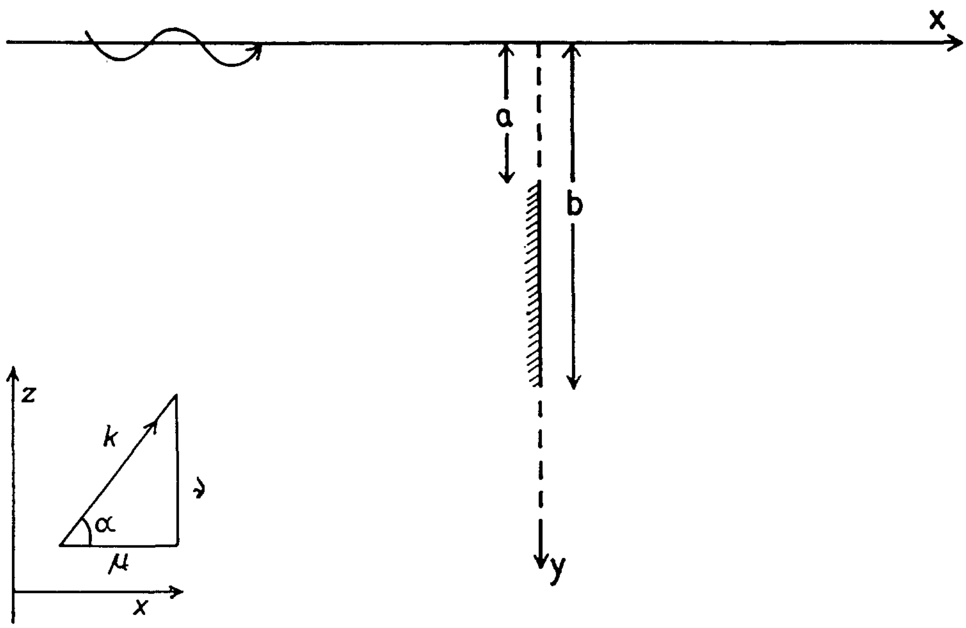

Fig. 1. Sketch of the problem.

Here the y-axis is taken vertically downwards through the plate and the xz-plane is taken as the position of the mean free surface. The geometry of the problem allows the z-dependence to be eliminated by assuming the velocity potential in the form

$$\psi(x,y,z,t) = \mathrm{Re}\{\phi(x,y)\exp(i\nu z - i\sigma t)\}$$

throughout, where $\phi(x,y)$ satisfies the boundary value problem described by

$$(\nabla^2 - \nu^2)\phi = 0 \quad \text{for} \quad y \geq 0, \tag{2.1}$$
$$K\phi + \phi_y = 0 \quad \text{on} \quad y = 0, \tag{2.2}$$
$$\phi_x = 0, \quad x = 0, \quad y \in S, \tag{2.3}$$
$$r^{1/2}\nabla\phi \text{ is bounded as } r \to 0 \tag{2.4}$$

where r is the distance from a submerged end of the plate,

$$\nabla\phi \to 0 \quad \text{as} \quad y \to \infty \tag{2.5}$$

and

$$\phi(x,y) = \begin{cases} T\exp(-Ky + i\mu x) & \text{as} \quad x \to \infty, \\ \exp(-Ky + i\mu x) + R\exp(-Ky - i\mu x) & \text{as} \quad x \to -\infty \end{cases} \tag{2.6}$$

where R and T are the (complex) reflection and transmission coefficients, respectively, and are to be obtained.

3. Method of solution

A solution for $\phi(x,y)$ satisfying (2.1), (2.2), (2.5) and (2.6) can be represented as

$$\phi(x,y)=\begin{cases} T\exp(-Ky+i\mu x)+\int_0^\infty A(k)L(k,y)\exp(-k_1x)\,\mathrm{d}k & \text{for} \quad x>0, \\ \exp(-Ky+i\mu x)+R\exp(-Ky+i\mu x)+ & \\ \qquad \int_0^\infty B(k)L(k,y)\exp(k_1x)\,\mathrm{d}k \quad \text{for} \quad x<0 & \end{cases} \tag{3.1}$$

where $k_1=(k^2+\nu^2)^{1/2}$ with $k_1=k$ when $\nu=0$ and $L(k,y)=k\cos ky-K\sin ky$.
Let

$$f(y)=\phi_x(0,y), \quad 0<y<\infty \tag{3.2}$$

and

$$g(y)=\phi(+0,y)-\phi(-0,y), \quad 0<y<\infty, \tag{3.3}$$

then

$$f(y)=0 \quad \text{for} \quad y\in S \tag{3.4}$$

and

$$g(y)=0 \quad \text{for} \quad y\in \bar{S}=(0,\infty)-S \tag{3.5}$$

The constants T, R and the functions $A(k), B(k)$ can be represented in terms of $f(y)$ and $g(y)$. This is done as follows.

Since ϕ_x is continuous across $x=0$, we obtain from (3.1) and (3.2) that

$$\begin{aligned} f(y) &= \phi_x(\pm 0,y)=i\mu T\exp(-Ky)-\int_0^\infty k_1A(k)L(k,y)\,\mathrm{d}k \\ &= i\mu(1-R)\exp(-Ky)+\int_0^\infty k_1B(k)L(k,y)\,\mathrm{d}k, \quad y>0. \end{aligned} \tag{3.6}$$

Utilizing Havelock's [3] inversion theorem and noting (3.4), we find from (3.6) that

$$T=1-R=-\frac{2iK}{\mu}\int_{\bar{S}} f(y)\exp(-Ky)\,\mathrm{d}y, \tag{3.7a}$$

$$A(k)=-B(k)=-\frac{2}{\pi}\frac{1}{k_1(k^2+K^2)}\int_{\bar{S}} f(y)L(k,y)\,\mathrm{d}y. \tag{3.7b}$$

Again, using (3.1) in (3.3) we find

$$g(y)=(T-R-1)\exp(-Ky)+\int_0^\infty \{A(k)-B(k)\}L(k,y)\,\mathrm{d}k. \tag{3.8}$$

Again, by utilizing Havelock's [3] inversion and noting (3.5), (3.7), we obtain from (3.8) that

$$R=-K\int_S g(y)\exp(-Ky)\,\mathrm{d}y, \tag{3.9a}$$

$$A(k) = \frac{1}{\pi}\frac{1}{k^2+K^2}\int_S g(y)L(k,y)\,\mathrm{d}y. \tag{3.9b}$$

Use of the condition (2.3) in the form

$$\lim_{x\to\pm 0}\phi_x(x,y) = 0, \quad y \in S$$

along with (3.1) and (3.9b) produces an integral equation for $g(y)$ as

$$\lim_{\epsilon\to 0}\int_S g(u)M(y,u;\epsilon)\,\mathrm{d}u = \pi i\mu(1-R)\exp(-Ky) \quad \text{for} \quad y \in S \tag{3.10}$$

where

$$M(y,u;\epsilon) = \int_0^\infty \frac{k_1 L(k,y)L(k,u)\exp(-\epsilon k_1)}{k^2+K^2}\,\mathrm{d}k \tag{3.11}$$

so that $M(y,u;\epsilon)$ is symmetric in y and u.

Again, as $\phi(x,y)$ is continuous across the gaps, use of (3.1) along with (3.7b) produces an integral equation for $f(y)$ as

$$\int_{\bar{S}} f(u)N(y,u)\,\mathrm{d}u = -\frac{1}{2}\pi R\exp(-Ky) \quad \text{for} \quad y \in \bar{S} \tag{3.12}$$

where

$$N(y,u) = \int_0^\infty \frac{L(k,y)L(k,u)}{k_1(k^2+K^2)}\,\mathrm{d}k \tag{3.13}$$

so that $N(y,u)$ is symmetric in y and u.

If we let

$$F(y) = -\frac{2}{\pi R}f(y) \quad \text{for} \quad y \in \bar{S}, \tag{3.14}$$

$$G(y) = \frac{1}{\pi i\mu(1-R)}g(y) \quad \text{for} \quad y \in S, \tag{3.15}$$

then $G(y)$ and $F(y)$ satisfy the integral equations

$$\lim_{\epsilon\to 0}\int_S G(u)M(y,u;\epsilon)\,\mathrm{d}u = \exp(-Ky) \quad \text{for} \quad y \in S \tag{3.16}$$

and

$$\int_{\bar{S}} F(u)N(y,u)\,\mathrm{d}u = \exp(-Ky) \quad \text{for} \quad y \in \bar{S} \tag{3.17}$$

It may be noted that the functions $G(y)$ and $F(y)$ in (3.16) and (3.17), respectively, must be real.

The relations (3.7a) and (3.9a) are now recast as

$$\int_{\bar{S}} F(y)\exp(-Ky)\,\mathrm{d}y = C, \tag{3.18}$$

and

$$\int_S G(y)\exp(-Ky)\,\mathrm{d}y = \frac{1}{\pi^2 K^2 C} \tag{3.19}$$

where

$$C = \frac{1-R}{i\pi R\sec\alpha} \tag{3.20}$$

It is important to note that C is real.

4. Upper and Lower Bounds for 'C'

Following Evans and Morris [13], we define an inner product

$$\langle f, g\rangle = \int_S f(y)g(y)\,\mathrm{d}y. \tag{4.1}$$

Then obviously $\langle f, g\rangle$ is symmetric and linear. Also, the operator $\mathcal{M}$ defined by

$$(\mathcal{M}f)(y) = \langle M(y,u;\epsilon), f(u)\rangle \tag{4.2}$$

is linear, self-adjoint and positive semi-definite.

As in Evans and Morris [13] for the solution of (3.16) we choose a one-term approximation as

$$G(y) \approx a_1 g_1(y) \tag{4.3}$$

where a_1 is a constant and $g_1(y)$ is to be chosen suitably.

Then

$$a_1 = \frac{\langle g_1(y), \exp(-Ky)\rangle}{\langle g_1(y), (\mathcal{M}g_1)(y)\rangle}. \tag{4.4}$$

Hence, from (3.19):

$$\frac{1}{\pi^2 K^2 C} = \langle G(y), \exp(-Ky)\rangle \geq \langle a_1 g_1(y), \exp(-Ky)\rangle$$

by using the same argument as [13].

Thus

$$C \leq A_0 \tag{4.5}$$

where

$$A_0 = \frac{\displaystyle\int_S g_1(y)\left[\lim_{\epsilon\to 0}\int_S\left\{\int_0^\infty \frac{k_1\exp(-\epsilon k_1)}{k^2+K^2}L(k,y)L(k,u)\,\mathrm{d}k\right\}g_1(u)\,\mathrm{d}u\right]\mathrm{d}y}{\pi^2 K^2\left[\displaystyle\int_S g_1(y)\exp(-Ky)\,\mathrm{d}y\right]^2} \tag{4.6}$$

Again, if we define another inner product by

$$\{f, g\} = \int_{\bar{S}} f(y)g(y)\,\mathrm{d}y \tag{4.7}$$

and another operator $\mathcal{N}$ by

$$(\mathcal{N}f)(y) = \{N(y,u), f(u)\}, \tag{4.8}$$

then it is obvious that $\{f, g\}$ is linear, symmetric and also the operator $\mathcal{N}$ is linear, self-adjoint and positive semi-definite.

If we choose a one-term approximation of $F(y)$ as

$$F(y) \approx b_1 f_1(y) \tag{4.9}$$

where b_1 is a constant and $f_1(y)$ is to be chosen suitably, then

$$b_1 = \frac{\{f_1(y), \exp(-Ky)\}}{\{f_1(y), (\mathcal{N}f_1)(y)\}} \tag{4.10}$$

Thus, by using (3.18) and the same argument as before, we find

$$C \geq B_0 \tag{4.11}$$

where

$$B_0 = \frac{\left[\int_{\bar{S}} f_1(y)\exp(-Ky)\,\mathrm{d}y\right]^2}{\int_0^\infty \frac{1}{k_1(k^2+K^2)}\left[\int_{\bar{S}} f_1(y)L(k,y)\,\mathrm{d}y\right]^2 \mathrm{d}k}. \tag{4.12}$$

Hence for the unknown real constant C, we find

$$B_0 \leq C \leq A_0 \tag{4.13}$$

where A_0 and B_0 are given by (4.6) and (4.12) respectively. Thus upper and lower bounds for $|R|$ and $|T|$ are obtained as

$$R_1 \leq |R| \leq R_2, \qquad T_1 \leq |T| \leq T_2 \tag{4.14}$$

where

$$R_1 = \frac{1}{(1+\pi^2 A_0^2 \sec^2\alpha)^{1/2}}, \qquad R_2 = \frac{1}{(1+\pi^2 B_0^2 \sec^2\alpha)^{1/2}}; \tag{4.15}$$

$$T_1 = \frac{\pi B_0 \sec\alpha}{(1+\pi^2 A_0^2 \sec^2\alpha)^{1/2}}, \qquad T_2 = \frac{\pi A_0 \sec\alpha}{(1+\pi^2 B_0^2 \sec^2\alpha)^{1/2}}. \tag{4.16}$$

5. Functions $g_1(y)$ and $f_1(y)$

The functions $g_1(y)$ and $f_1(y)$ are chosen as the explicit solutions of the appropriate integral equations for the problem of water wave diffraction by a thin, vertical plate sub-merged in deep water for the case of normal incidence and are given (cf. [17–19]) by:

$$g_1(y) = A_1\lambda(y) \qquad (A_1 \neq 0) \tag{5.1}$$

and

$$f_1(y) = B_1\chi'(y) \qquad (B_1 \neq 0) \tag{5.2}$$

where

$$\lambda(y) = \exp(-Ky)\int_a^y \frac{(d^2-u^2)\exp(Ku)}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}\,du, \quad a<y<b \tag{5.3}$$

and

$$\chi(y) = \begin{cases} \exp(-Ky)\displaystyle\int_a^y \frac{(d^2-u^2)\exp(Ku)}{\{(a^2-u^2)(b^2-u^2)\}^{1/2}}\,du, & 0<y<a, \\ -\exp(-Ky)\displaystyle\int_b^y \frac{(d^2-u^2)\exp(Ku)}{\{(u^2-a^2)(u^2-b^2)\}^{1/2}}\,du, & y>b \end{cases} \tag{5.4}$$

with d^2 given by

$$\int_a^b \frac{(d^2-u^2)\exp(Ku)}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}\,du = 0 \tag{5.5}$$

Substituting these in the expressions (4.6) and (4.12), A_0 and B_0 are obtained as

$$A_0 = \frac{4}{\pi^2\gamma_0^2}\int_0^\infty \frac{k_1}{k^2+K^2}\left[\int_a^b \frac{(d^2-u^2)\sin ku}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}\,du\right]^2 dk \tag{5.6}$$

and

$$B_0 = \frac{\frac{1}{4}(\alpha_0-\beta_0)^2}{\displaystyle\int_0^\infty \frac{k^2}{k_1(k^2+K^2)}\left[\int_a^b \frac{(d^2-u^2)\sin ku}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}\,du\right]^2 dk} \tag{5.7}$$

where

$$\alpha_0 = \int_{-a}^a \frac{(d^2-u^2)\exp(-Ku)}{\{(a^2-u^2)(b^2-u^2)\}^{1/2}}\,du, \tag{5.8}$$

$$\beta_0 = \int_b^\infty \frac{(d^2-u^2)\exp(-Ku)}{\{(u^2-a^2)(u^2-b^2)\}^{1/2}}\,du, \tag{5.9}$$

and

$$\gamma_0 = \int_a^b \frac{(d^2-u^2)\exp(-Ku)}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}\,du. \tag{5.10}$$

It is to be noted that in the numerator of A_0 in (4.6), after $g_1(u)$ is substituted from (5.1), the limit can be taken inside the k-integral so as to produce (5.6) ultimately. Again, it may be noted that for $\alpha = 0$,

$$A_0 = B_0 = \frac{\alpha_0-\beta_0}{\pi\gamma_0}$$

giving an exact value of C for $\alpha = 0$, so that in this case $R = i\gamma_0/\Delta$, where $\Delta = \alpha_0-\beta_0-i\gamma_0$, which agrees with the result obtained by Evans [9].

Table 1. Lower and upper bounds for the reflection coefficient $|R|$ for $a/b = 0.5$

	$\alpha = 15°$		$\alpha = 45°$		$\alpha = 75°$		$\alpha = 85°$	
K_b	R_1	R_2	R_1	R_2	R_1	R_2	R_1	R_2
0.05	0.000 442	0.000 442	0.000 324	0.000 324	0.000 118	0.000 118	0.000 040	0.000 040
0.4	0.016 985	0.017 043	0.012 410	0.012 454	0.004 534	0.004 551	0.001 527	0.001 532
0.8	0.037 454	0.037 489	0.027 281	0.027 310	0.009 937	0.009 950	0.003 344	0.003 348
1.6	0.045 582	0.045 591	0.032 609	0.032 653	0.011 684	0.011 724	0.003 924	0.003 938
2.4	0.032 469	0.032 492	0.022 468	0.022 636	0.007 843	0.007 974	0.002 625	0.002 673
3.0	0.021 963	0.022 009	0.014 749	0.014 993	0.005 044	0.005 215	0.001 685	0.001 746

6. Numerical results

The expressions(5.6) and (5.7) and hence the upper and lower bounds for $|R|$ and $|T|$ are evaluated numerically for various values of the parameters $Kb, a/b$ and α. The various single integrals appearing here are evaluated by using a ninety-six-point Gauss quadrature formula. For the repeated integrals the inner integrals are evaluated by a ninety-six-point Gauss quadrature formula, while the outer integrals over $(0, \infty)$ are split into those over (0,1) and $(1, \infty)$. The integrals over (0,1) are computed by ninety-six-point Gauss quadrature. For

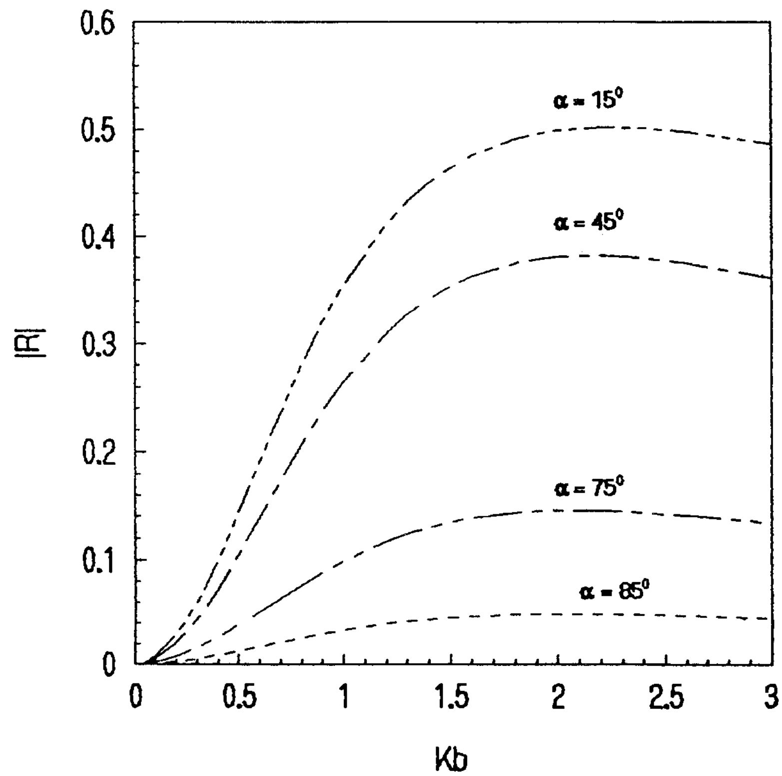

Fig. 2. Reflection coefficient for a/b = 0.05.

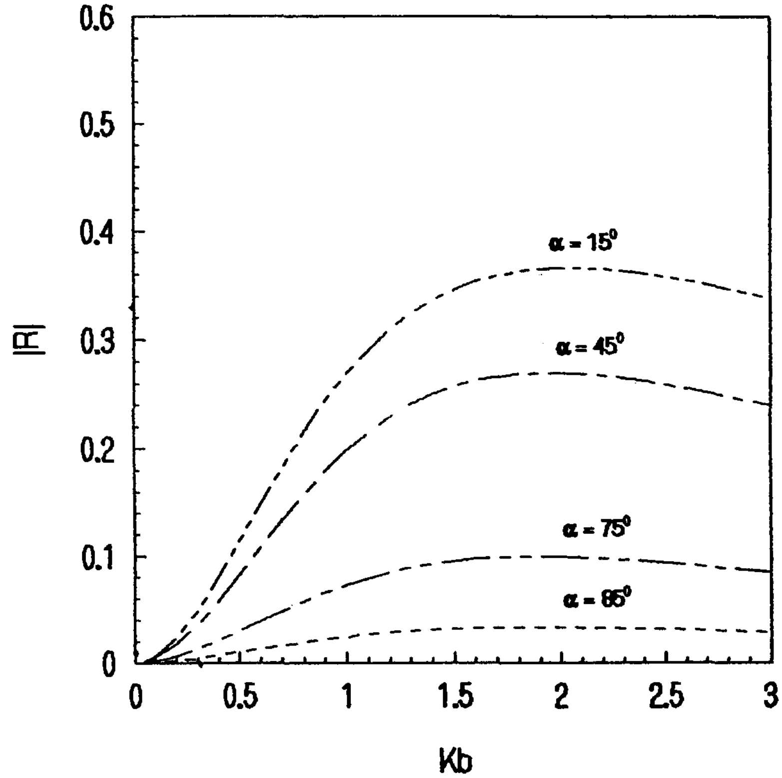

Fig. 3. Reflection coefficient for a/b = 0.1.

the integrals over $(0, \infty)$ the interval is replaced by $(1, X)$ where X is a large number chosen suitably so as to obtain the values of the integrals correct up to some desired number of decimal places by using Simpson's rule. Some representative results for the lower and upper bounds R_1 and R_2 for $|R|$ are given in Table 1 for $a/b = 0.5$ and $\alpha = 15°, 45°, 75°, 85°$.

It is observed from this table that the two bounds for $|R|$ coincide up to 3–5 decimal places for most cases and as such the true values of $|R|$ are obtained correct up to 3–5 decimal places. The same is also true for the two bounds for $|T|$, which are, however, not tabulated here.

In Figures 2 and 3, $|R|$ is depicted against the wave number Kb for $a/b = 0.05$ and 0.1, respectively, and $\alpha = 15°, 45°, 75°, 85°$. Since the two bounds R_1 and R_2 of $|R|$ for any value of Kb are very close, we have taken their average while drawing these figures. From Table 1 and from these figures it is observed that for fixed Kb and $a/b, |R|$ decreases with increasing angle of incidence α. For $\alpha = 85°, |R|$ becomes very small for any, a/b and Kb. As $\alpha \to 90°$, it is noted that the two bounds R_1 and R_2, and hence $|R|$, tends to zero. This is obvious since the incident wave then almost grazes along the plate. Again, for fixed, a/b and $\alpha, |R|$ increases with the increase of Kb until a maximum is reached and then decreases to zero for further increase in Kb. This is also plausible since for large Kb the wavelength becomes small and the waves are confined within a thin layer near the free surface and almost total transmission then occurs since the presence of the plate is practically not felt by these waves.

7. Conclusion

An approximate method based on a one-term Galerkin approximation to the solutions of certain integral equations has been used to obtain very accurate upper and lower bounds for the reflection and transmission coefficients in the problem of diffraction of an obliquely incident train of surface waves by a thin vertical plate submerged in deep water. For the case of normal incidence, the upper and lower bounds coincide, giving rise to the exact, known result. In our computer program for the numerical evaluation of the upper and lower bounds for a general value of α (the angle of incidence), if α is set equal to zero, the numerical values for the upper and lower bounds practically coincide. This gives a check of the correctness of out computer program. This method can also be applied to obtain approximate results for other problems involving barriers with a single gap or a number of gaps in deep water.

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